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ABSTRACT

Although an analysis of covariance (ANCOVA) allows for the removal of an uncontrolled source of variation that is represented by the covariates, this "correction," which occurs with the dependent variable scores is unfortunately seen by some as a blanket adjustment device that can be used with an inadequate amount of consideration for the homogeneity of slopes assumption. When regression slopes are found not to be parallel, treatment effects will most likely be biased, and there will be a reduction in the efficiency of the analysis. Twenty heuristic data sets coupled with analysis of variance and ANCOVA analyses are provided to illustrate what may occur when the homogeneity of slopes requirement is not met. Even though each of the groups had identical means, variations in the distribution of data for one of the groups studies led to varying slopes. Consequently, three different ANCOVA values resulted, only one of which was accurate. It should be noted that the homogeneity of slopes assumption can be violated to some degree without seriously affecting the robustness of tests of significance in ANCOVA. (Contains 6 tables, 23 figures, and 3 references.) (Author/SLD)

Testing for Homogeneity of Slopes in Analysis of Covariance: A Tutorial

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Abstract

Although an analysis of covariance allows for the removal of an uncontrolled source of variation that is represented by the covariate(s), this "correction", which occurs with the dependent variable scores is unfortunately seen by some as a blanket adjustment device that can be used with an inadequate amount of consideration for the homogeneity of slopes assumption. When regression slopes are found to be unparallel, treatment effects will most likely be biased and there will be a reduction in the efficiency of the analysis. Heuristic data sets coupled with ANOVA and ANCOVA analyses are provided to illustrate what may occur when the homogeneity of slopes requirement is not met.

Testing for Homogeneity of Slopes
in Analysis of Covariance: A Tutorial

In order to obtain estimates of treatment effects that are unbiased one can statistically remove variance from the dependent variable by analysis of covariance (ANCOVA). This method of statistical control combines regression analysis with sums-of-squares analysis of variance which is computed on the adjusted scores of the dependent variable. Kirk (1995) suggests using statistical control as a means of removing possible sources of bias from an experiment when such bias is difficult to remove by experimental control. In this way, the researcher is better able to achieve unbiased estimates of treatment effects and a reduction in experimental error. Since ANCOVA allows for the removal of an uncontrolled source of variation that is represented by the covariate(s), the researcher stands the chance of benefitting from an increase in power which translates into a greater possibility of finding an effect, if one does indeed exist.

In order to conduct an analysis of covariance, certain assumptions must be met:

1. The dependent variable and covariate(s) should be correlated.
2. The independent variable(s) and the covariate(s) should be uncorrelated.
3. The residualized dependent variable for each level of the independent variable should be normally distributed and

equality should exist between the variances of the residualized dependent variable for each level of the independent variable.

4. The adjustment which occurs as a result of manipulation of the covariate should be unrelated to the goals of the experiment.
5. Parallel regression slopes should exist between the covariate and the dependent variable (Kirk, 1995; Levy, 1980; Loftin & Madison, 1991).

One problem, however, is that this "correction", which occurs with the dependent variable scores is unfortunately seen by some as a blanket adjustment device that can be used with an inadequate amount of consideration for the assumptions that make for a successful ANCOVA (Loftin & Madison, 1991). In actuality, there are many occasions when data sets do not meet the requirements necessary to make an appropriate adjustment. The assumption of homogeneity of slopes is the focus of this paper. Although ANCOVA can aid in the prediction of error variance that is associated with the dependent variable, it is essential to remember that when regression slopes are found to be unparallel, treatment effects will most likely be biased and there will be a reduction in the efficiency of the analysis. Loftin & Madison (1991) assert that the homogeneity of slopes assumption is often neglected and that this is where most researchers fail in terms of obtaining accurate ANCOVA analyses. By not first checking to be sure that the experimental groups have equal regression slopes, researchers may find themselves facing biased results in the form of probability statements which are lacking in validity.

As an illustration of what may occur when the homogeneity of slopes requirement is not met, heuristic data sets (N=20) are provided depicting three possible conditions: 1) parallel regression slopes among the two levels of the independent variable, 2) unparallel regression slopes, and 3) regression slopes occurring as a result of one of the data sets having truncated range. In these examples, two groups (A and B), each consisting of ten cases, will be presented; Group A will be held constant throughout each of the examples while the data in Group B will be manipulated. For each of the examples, the dependent variable, Y, will represent undergraduate grade point average (GPA), the independent variable will be group membership, and the covariate, X, will represent American College Test (ACT) scores. The X and Y means for the two groups, A and B, will be held constant: $\overline{X_a} = 18$, $\overline{Y_a} = 2.0$, $\overline{X_b} = 25$, $\overline{Y_b} = 3.0$.

In the first example (see Table 1), the data represent a case in which the slope for the regression of Group A (b_a =.15) is nearly equal to the slope for the regression of Group B (b_b =.22). This is an example where the homogeneity of slopes assumption is not violated (see Figure 1). Looking at the unadjusted cell means, it can be seen that the difference in mean scores for Group A and B is statistically significant, $F(1, 18) = 8.65$, $p = .009$. However, adjusted cell means resulting from the ANCOVA show that most of the variance is attributed to the covariate, ACT scores.

Consequently, the difference in adjusted cell means is not statistically significant, $F(1,17) = .41$, $p = .53$ (see Table 2).

Example 2 (see Table 3 and Figure 2) depicts a situation in which the homogeneity of slopes assumption has been violated. Here, the case means are exactly the same as those found in Example 1, however, the slope for the regression of Group A ($b_a = .15$) is different than the slope for the regression of Group B ($b_b = -.10$). The data provided illustrates that the difference in the unadjusted cell means is statistically significant when an ANOVA is conducted; $F(1,18) = 10.61$, $p = .004$ (see Table 4). In this example, the difference in the adjusted cell means as calculated by the ANCOVA, remains statistically significant at the .05 level, $F(1,17) = 5.03$, $p = .04$ (see Table 4).

The last example (see Table 5 and Figure 3) also represents a situation in which the homogeneity of slopes assumption has been violated. In this particular case, the data for Group B exhibit a truncated range, which may often be the case in many data sets. Case means remain identical to those found in Example 1 and Example 2. As found in Example 2, the slope for the regression of Group A ($b_a = .15$) is different than the slope for the regression of Group B ($b_b = -.03$). Data supplied when an ANOVA is performed demonstrates that the difference in the unadjusted cell means is statistically significant; $F(1, 18) = 15.63$, $p = .001$ (see Table 5). It is illustrated that when an ANCOVA is performed, the difference in the adjusted cell means is not statistically significant; $F(1, 17) = .11$, $p = .75$.

As indicated in the examples above, failing to check the homogeneity of regression slopes can lead to erroneous interpretation of results. Even though each of the groups had identical means, variations in the distribution of Group B data led to varying slopes. Consequently, three different ANCOVA values resulted; only one of which was accurate. It should be noted that the homogeneity of slopes assumption can be violated to some degree without seriously affecting the robustness of tests of significance in the analysis of covariance. Loftin & Madison (1991) suggest that the use of regression lines that are not perfectly homogeneous can be used appropriately if the sample sizes of all groups are balanced or beta weights within individual groups differ by less than .4. It should also be noted that Kirk (1995) suggests using an F-test to check for the homogeneity of regression coefficients. He further recommends using a numerically large level of significance such as .10 to .25, in order to reduce the risk of accepting the hypothesis of homogeneity of regression coefficients when the hypothesis is actually false. In other words, choosing a larger level of significance will lessen the chances of committing a type II error.

References

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Loftin, Lynn B., & Madison, Susan Q. (1991). The extreme dangers of covariance corrections. In Thompson, Bruce (Ed.), Advances in educational research: Substantive findings, methodological developments (vol. 1, pp.133-147). Greenwich, CT: Jai Press Incorporated.

EXAMPLE 1

Table 1. Data set and means.

Group A	Case	X_A (ACT)	Y_A (GPA)
	1	12	1.2
	2	13	1.6
	3	14	1.0
	4	16	1.2
	5	17	2.4
	6	20	1.8
	7	20	2.8
	8	21	2.2
	9	23	2.6
	10	24	3.2
		$\bar{X}_A = 18.0$	$\bar{Y}_A = 2.0$

Group B	Case	X_B (ACT)	Y_B (GPA)
	1	21	1.8
	2	22	3.2
	3	23	2.2
	4	24	2.6
	5	24	2.2
	6	25	3.2
	7	26	3.8
	8	27	3.2
	9	28	4.0
	10	30	3.8
		$\bar{X}_B = 25.0$	$\bar{Y}_B = 3.0$

EXAMPLE 1

Table 2.

ANOVA of GPA by Group.

Source	df	SS	MS	F	p_{calc}
Group	1	5.00	5.00	8.65	.009
Error	18	10.40	.58		

ANCOVA of GPA by Group with ACT as covariate.

Source	df	SS	MS	F	p_{calc}
ACT	1	6.68	6.68	30.54	.000
Group	1	.09	.09	.41	.529
Error	17	3.72	.22		

EXAMPLE 1

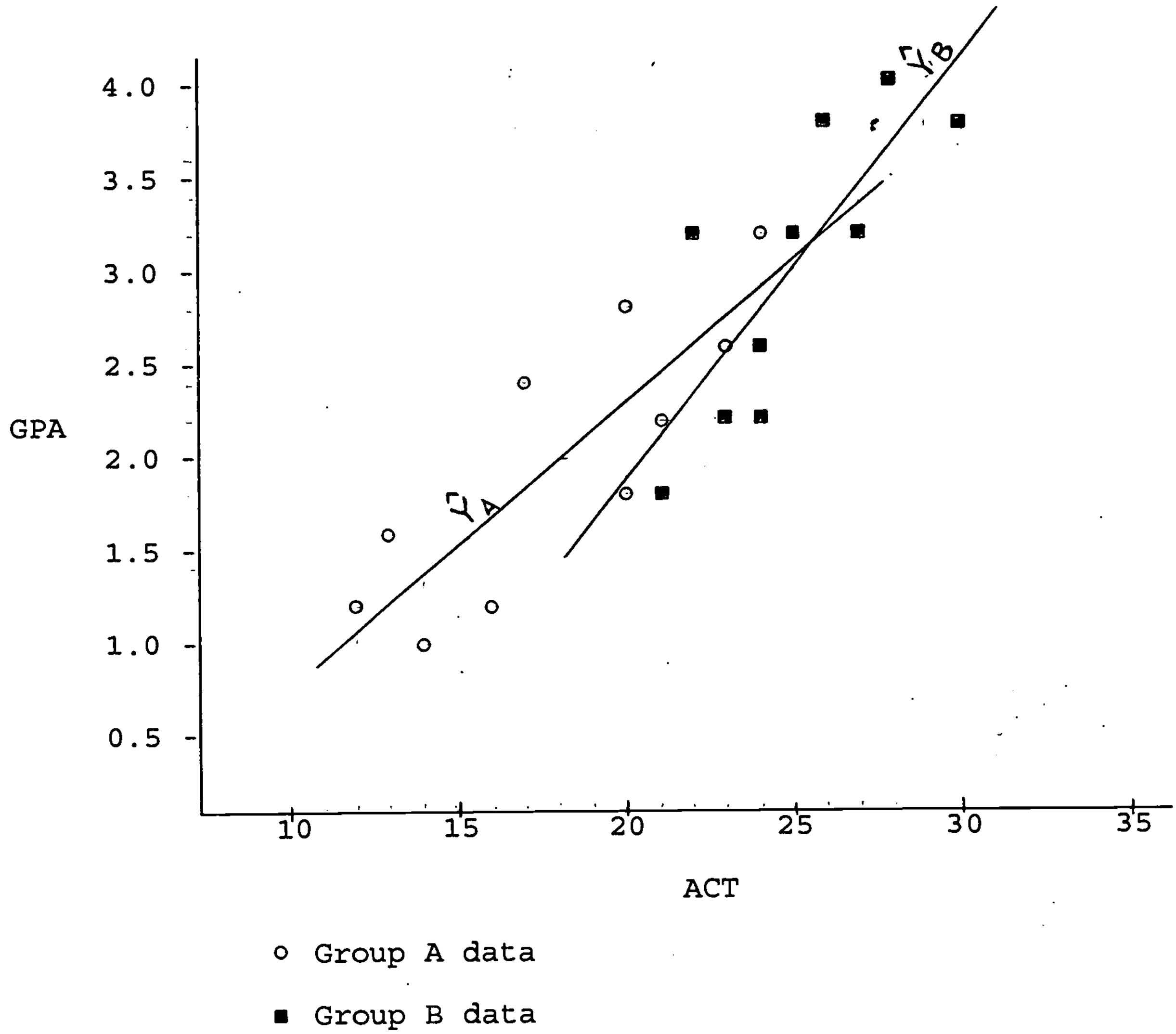

Figure 1. Scatter Plot of Group A and B data with regression lines.

EXAMPLE 2

Table 3. Data set and means.

Group A	Case	X_A (ACT)	Y_A (GPA)
	1	12	1.2
	2	13	1.6
	3	14	1.0
	4	16	1.2
	5	17	2.4
	6	20	1.8
	7	20	2.8
	8	21	2.2
	9	23	2.6
	10	24	3.2
		$\bar{X}_A = 18.0$	$\bar{Y}_A = 2.0$

Group B	Case	X_B (ACT)	Y_B (GPA)
	1	18	3.4
	2	19	3.8
	3	21	3.2
	4	23	3.8
	5	25	2.6
	6	26	3.4
	7	27	2.4
	8	30	2.6
	9	30	2.8
	10	31	2.0
		$\bar{X}_B = 25.0$	$\bar{Y}_B = 3.0$

EXAMPLE 2

Table 4.

ANOVA of GPA by Group.

Source	df	SS	MS	F	p_{calc}
Group	1	5.00	5.00	10.61	.004
Error	18	8.48	.47		

ANCOVA of GPA by Group with ACT as covariate.

Source	df	SS	MS	F	p_{calc}
ACT	1	.05	.05	.10	.756
Group	1	2.49	2.49	5.03	.039
Error	17	8.43	.496		

EXAMPLE 2

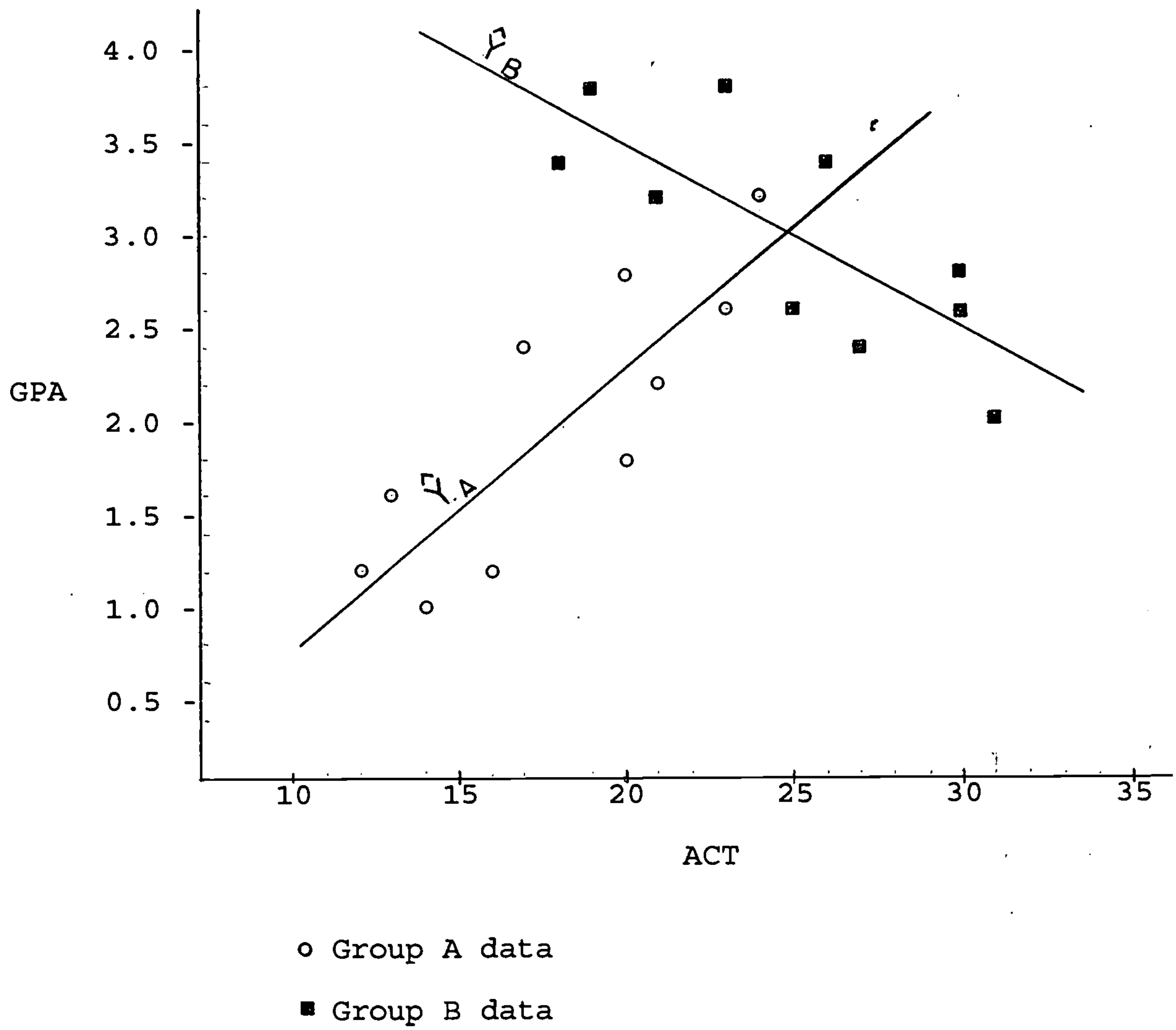

Figure 2. Scatter Plot of Group A and B data with regression lines.

EXAMPLE 3

Table 5. Data set and means.

Group A	Case	X_A (ACT)	Y_A (GPA)
	1	12	1.2
	2	13	1.6
	3	14	1.0
	4	16	1.2
	5	17	2.4
	6	20	1.8
	7	20	2.8
	8	21	2.2
	9	23	2.6
	10	24	3.2
		$\bar{X}_A = 18.0$	$\bar{Y}_A = 2.0$

Group B	Case	X_B (ACT)	Y_B (GPA)
	1	23	3.2
	2	23	3.0
	3	24	2.6
	4	24	3.4
	5	25	2.6
	6	25	3.2
	7	26	3.0
	8	26	3.2
	9	27	3.0
	10	27	2.8
		$\bar{X}_B = 25.0$	$\bar{Y}_B = 3.0$

EXAMPLE 3

Table 6.

ANOVA of GPA by Group.

Source	df	SS	MS	F	p_{calc}
Group	1	5.00	5.00	15.63	.001
Error	18	5.76	.32		

ANCOVA of GPA by Group with ACT as covariate.

Source	df	SS	MS	F	p_{calc}
ACT	1	3.04	3.04	19.02	.000
Group	1	.02	.02	.11	.747
Error	17	2.72	.16		

EXAMPLE 3

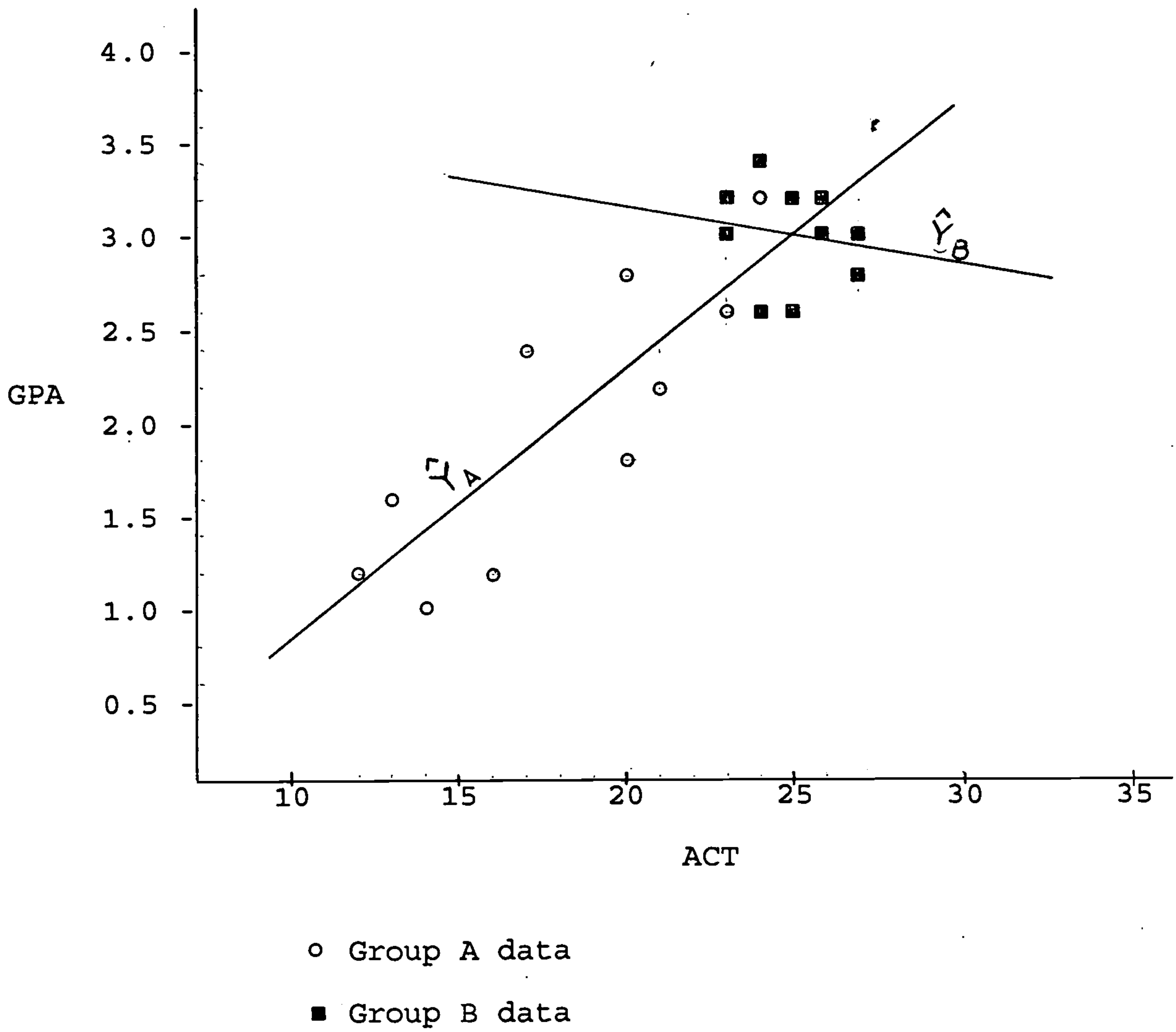

Figure 3. Scatter Plot of Group A and B data with regression lines.

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